Završni ispit iz Kvantnih računala (6. veljače 2019.)

Ime, prezime i JMBAG:

Uputa:

- Ispit se sastoji od 10 zadataka najčešće u obliku pitanja s ponuđenim odgovorima.
- Odgovore koje smatrate točnima označite (zacrnite) na posebnom obrascu. Mogu se pojaviti zadaci u kojima je potrebno označiti više od jednog ponuđenog odgovora.
- U praznom prostoru pored zadatka ili na dodatnim papirima napišite obrazloženje ili računski postupak koji vas je doveo do rješenja koje smatrate točnim.
- Točno riješeni zadatak donosi 4 boda. Kazneni (negativni) bodovi se ne obračunavaju.

Notacija i terminologija:

- Vektori $|0\rangle = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$ i $|1\rangle = \left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$ čine ortonormiranu bazu u $\mathcal{H}^{(2)}$.
- Pri realizaciji qubita stanjima polarizacije fotona, vektori $|0\rangle = |x\rangle$ i $|1\rangle = |y\rangle$ odgovaraju stanjima linearne polarizacije u x-smjeru i u y-smjeru, bazu $\{|x\rangle, |y\rangle\}$ obilježavamo simbolom $\bigoplus$, a bazu $\{\frac{1}{\sqrt{2}}(|x\rangle \pm |y\rangle)\}$ obilježavamo simbolom $\bigotimes$.
- Pri realizaciji qubita projekcijom spina čestice spinskog kvantnog broja $s = {}^1\!/_2$ na z-os uzimamo da $|0\rangle$ i $|1\rangle$ odgovarju projekcijama $\hbar/2$ i $-\hbar/2$.
- Računalnu bazu u prostoru stanja dvaju qubitova obilježavamo s $\{|ij\rangle = |i\rangle \otimes |j\rangle\,; i, j = 0, 1\}$.

1 Operator $S = R[\pi/2]$ u računalnoj bazi ima matrični prikaz: // *In the computational basis, the matrix representation of the operator* $S = R[\pi/2]$ *is:*

$$\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i} \end{pmatrix}$$

Koje od navedenih tvrdnji su istinite? // *Which of the following statements are true?*

(a) S je unitaran operator. // S *is a unitary operator.* **točno**

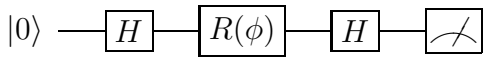
(b) S je hermitski operator. // S *is a Hermitean operator.*

(c) Na Blochovoj sferi, S rotira stanje kvantnog bita za π oko x-osi. // *On the Bloch sphere,* S *rotates the state of a qubit by* π *about the* x*-axis*

(d) S rotira stanje za $\pi/2$ oko z-osi. // S *rotates the state by* $\pi/2$ *about* z*-axis.* **točno**

(e) S rotira stanje za $\pi/2$ oko x-osi. // S *rotates the state by* $\pi/2$ *about* x*-axis.*

2 Razmatramo kvantni logički krug // *Consider the following quantum logical circuit*



Kolika je vjerojatnost da u mjerenju dobijemo vrijednost 1 (tj. da qubit bude izmjeren u stanju $|1\rangle$)? // *What is the probability that in the measurement we get the value* 1 *(ie. that the qubit is in the state* $|1\rangle$*)?*

(a) 0

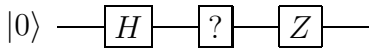
(b) $\frac{1}{2}(1 + \cos\phi)$

(c) $\frac{1}{2}(1 - \cos\phi)$ **točno**

(d) $\cos\phi$

(e) $\cos^2\phi$

3 Ako na izlazu iz kvantnog logičkog kruga // *If at the output of the quantum logical circuit*



dobivamo stanje // *the state is*

$$\frac{1}{\sqrt{2}}\big(\,|0\rangle - \mathrm{i}\,|1\rangle\,\big),$$

operator označen upitnikom je // *the operator indicated with the question mark is*

(a) X

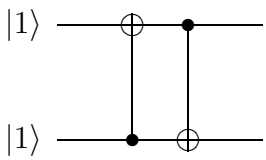
(b) Y

(c) Z

(d) S **točno**

(e) T

4 Stanje sustava na izlaznoj strani kvantnog logičkog kruga // *The state of the system at the output of the quantum logical circuit*



je // *is*

(a) $|01\rangle$ **točno**

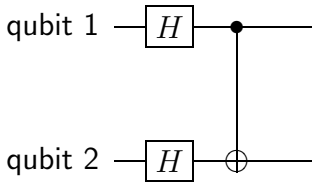
(b) $|10\rangle$

(c) $\frac{1}{\sqrt{2}}\big(\,|01\rangle + |10\rangle\,\big)$

(d) $\frac{1}{\sqrt{2}}\big(\,|01\rangle - |10\rangle\,\big)$

(e) $|11\rangle$

5 Shvatimo li kvantni logički krug // *If we consider the quantum logical circuit*



kao jedan operator, njegov matrični prikaz je: // *as a single operator, its matrix representation is:*

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 0 & 1 & 0 & -1\\ 1 & 0 & -1 & 0\end{pmatrix}$

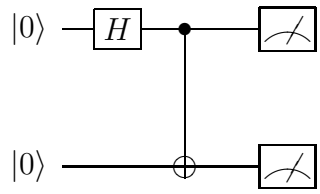
(b) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1\\ 0 & 1 & -1 & 0\end{pmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 0 & 1 & 1\end{pmatrix}$

(d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & -1 & 1\end{pmatrix}$

(e) $\frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\\ 1 & 1 & -1 & -1\end{pmatrix}$ **točno**

6 Kolika je vjerojatnost da na izlazu iz kvantnog logičkog kruga // *What is the probability that at the output of the quantum logical circuit*



sustav izmjerimo u stanju $|01\rangle$? // *the system is measured in the state* $|01\rangle$*?*

(a) 0 **točno**

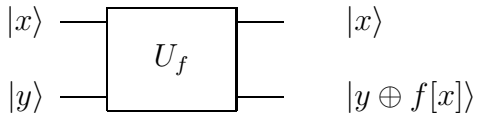
(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

(e) 1

7 Funkciju jednog bita $f : \{0,1\} \to \{0,1\}$ implementiramo unitarnim operatorom // *One-bit function* $f : \{0,1\} \to \{0,1\}$ *is implemented by the unitary operator*



$x, y \in \{0,1\}$

Ako je $f[x] = x$, operator U možemo odabrati kao: // *If* $f[x] = x$*, the operator* U *can be chosen as:*

(a) I

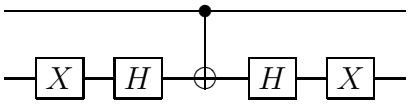
(b) $I \otimes X$

(c) cNOT **točno**

(d) $(I \otimes X) \cdot \mathrm{cNOT}$

(e) Ništa od navedenog. // *None of the above.*

8 Kvantni logički krug prikazan slikom // *The quantum logical circuit shown below*



jest implementacija operatora // *implements the operator*

$$U_f\,|x\rangle = \mathrm{e}^{\mathrm{i}\phi}(-1)^{f[x]}\,|x\rangle\,, \qquad \phi \in \mathbb{R}, \qquad x = 00, 01, 10, 11,$$

gdje je $f[x] = 0$ za svaki x osim za $x = w$, za koji vrijedi $f[w] = 1$. Odredi w. // *where* $f[x] = 0$ *for all* x *except for* $x = w$ *for which* $f[w] = 1$. *Find* w.

(a) $w = 00$

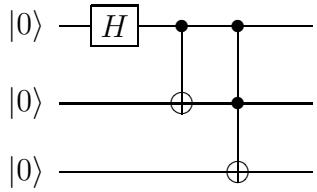
(b) $w = 01$

(c) $w = 10$ **točno**

(d) $w = 11$

(e) Ništa od navedenog (nema rješenja). // None of the above (no solution).

9 Na izlazu iz kvantnog logičkog kruga // *At the output of the quantum logical circuit*



stanje sustava je: // *the state of the system is:*

(a) $|000\rangle$

(b) $|111\rangle$

(c) $\frac{1}{\sqrt{2}}\big(\,|000\rangle + |100\rangle\,\big)$

(d) $\frac{1}{\sqrt{2}}\big(\,|000\rangle + |110\rangle\,\big)$

(e) $\frac{1}{\sqrt{2}}\big(\,|000\rangle + |111\rangle\,\big)$ **točno**

10 Pretražujemo li bazu veličine 10^6 Groverovim algoritmom, Groverov operator mora djelovati približno // *If a database of size* 10^6 *is searched using Grover's algorithm, the Grover's operator must operate approximately*

(a) 10 puta. // 10 *times.*

(b) 100 puta. // 100 *times.*

(c) 1000 puta. // 1000 *times.* **točno**

(d) 10^6 puta. // 10^6 *puta.*

(e) 2^6 puta. // 2^6 *puta.*